Can Hyperscalers Earn Back Their AI Infrastructure Investment?

Value Creation, Value Capture, and Investment Recovery*

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Abstract

The surge in artificial-intelligence (AI) infrastructure investment raises a financial question distinct from the productivity effects of AI: can the firms financing the buildout capture enough of the value created by AI as cash flow to recover their investment? This paper develops a discounted-cash-flow framework that separates economy-wide value creation from investment recovery by those firms. The framework is applied to the reported and projected 2024–2027 capital expenditures of Amazon, Alphabet, Microsoft, Meta, and Oracle. Under the baseline 20-year U.S. calibration, recovering the full reported 2024–2027 capital-expenditure benchmark requires the long-run level of GDP to be approximately 19 percent above its no-AI path under the baseline value-capture calibration. The required economic payoff rises when the capital-bearing value-capture rate is lower, diffusion is delayed, the required return is higher, the recovery horizon is shorter, or additional post-2027 expansion investment is included. It falls when a smaller fraction of reported capital expenditure enters the valuation, a larger economic base is used, or monetizable gains outside measured GDP are included. The results show that even a large economy-wide payoff does not ensure investment recovery: enough of the value created by AI must reach the firms financing the infrastructure as cash flow, and it must do so sufficiently quickly.

Keywords: Artificial intelligence; AI infrastructure investment; value capture; investment recovery; discounted cash flow

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1. Introduction

The surge in artificial-intelligence (AI) infrastructure investment has raised a simple financial question: can the hyperscalers financing the buildout recover their investment at the required rate of return, or will the current expansion ultimately prove to be an overinvestment in AI infrastructure? This paper develops a valuation framework for answering that question. The central issue is not whether AI can create economic value, but whether the firms financing data centers, servers and accelerators, networking equipment, and supporting power infrastructure can generate enough durable after-tax cash flow to recover the capital expenditure entering the valuation.

The paper examines potential overinvestment in AI infrastructure, a question distinct from stock-market mispricing. Conceptually, the relevant financial test compares investment attributable to the AI transition with the corresponding AI-attributable unlevered after-tax cash flow accruing to the same firms, with both measured relative to a common no-AI counterfactual.[1] Public disclosures do not identify the counterfactual investment and cash-flow paths needed to construct a strictly incremental net present value (NPV). The quantitative analysis therefore uses the full reported-and-projected 2024–2027 capital-expenditure path as the gross reported-capex benchmark. To account for the possibility that only part of this expenditure is attributable to the AI transition and incremental relative to the no-AI path, the sensitivity analysis allows only a fraction of the reported-and-projected capital expenditure to enter the investment benchmark. The firms included in the benchmark are Amazon, Alphabet, Microsoft, Meta, and Oracle, referred to throughout the paper as the capital-bearing firms.

Potential overinvestment in AI infrastructure need not imply an asset-price bubble or irrational firm behavior. Irreversibility and uncertainty generally increase the value of waiting before committing capital (Dixit and Pindyck, 1994). In AI infrastructure, however, firms may invest early if delaying could allow rivals to secure a lasting competitive advantage. Competition for a few dominant positions can therefore induce individually rational firms to overcommit capital relative to the socially efficient level (Rungcharoenkitkul, 2026). Such strategic motives can help explain the timing and scale of the infrastructure buildout, but they do not ensure that the resulting investment will earn its required return. This paper therefore takes the investment benchmark as given and asks how much cash flow must accrue to the capital-bearing firms for it to be recovered at the required rate of return.

The distinction between economy-wide gains from AI and recovery of AI infrastructure investment by the firms that finance it is central to the analysis. As a general-purpose technology, AI may generate gains throughout the economy as firms redesign workflows, accumulate complementary intangible capital, develop new products, and reorganize production. These adjustments can raise value added, reduce production costs, and improve matching and forecasting, but the resulting gains need not accrue to the capital-bearing firms. They may instead be retained by adopting firms, passed on to customers through lower prices, shared with workers and upstream suppliers, or realized as consumer surplus. Conversely, more modest economy-wide gains may support investment recovery if the capital-bearing firms can monetize a sufficiently large portion through infrastructure services and complementary assets such as cloud platforms, proprietary data, distribution channels, and established relationships with enterprise customers.[2] Investment recovery therefore depends on whether the monetizable value associated with AI is large enough, and whether a sufficient portion of that value becomes cash flow for the capital-bearing firms, to meet the investment-side requirement.

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[1] Section 2 defines the cash-flow measure and matches its accounting boundary to the capital expenditure included in the valuation.

[2] Scale economies, high fixed costs, vertical integration, switching frictions, and control over scarce complementary assets may strengthen the ability of the capital-bearing firms to capture part of the value generated downstream (Athey and Scott Morton, 2025; OECD, 2025). These features do not, however, imply complete extraction of economy-wide AI-related value or guarantee that revenue accruing to these firms will ultimately be retained as cash flow available for investment recovery.

The paper is closest to Wachter and Wachter (2026), who embed the same 2024–2027 capital-expenditure path in a two-sector model with rare productivity booms. Using a revealed-preference argument and assuming that marginal investment has zero net present value, they infer the AI-sector productivity boom consistent with the projected investment path and trace its implications for aggregate output. This paper addresses a complementary financial question. Their production-side analysis maps the investment path into the productivity and output payoff needed to rationalize the investment, whereas the financial analysis here additionally asks how much of the economy-wide monetizable value associated with AI must become cash flow accruing to the capital-bearing firms for the investment benchmark to earn its required return. Section 2 formalizes the connection between the two approaches.

The framework takes economy-wide AI-related value as an input rather than estimating it. It therefore complements the literature on general-purpose technologies, task-based automation, and AI adoption, which examines how technological change affects productivity, output, innovation, and the distribution of economic gains (Bresnahan and Trajtenberg, 1995; Acemoglu and Restrepo, 2018, 2019; Brynjolfsson, Rock, and Syverson, 2021; Babina et al., 2024; Korinek and Suh, 2024; Brynjolfsson, Li, and Raymond, 2025). The paper also draws on the literatures on appropriability, value appropriation, and platform competition. Appropriability regimes and control over complementary assets affect firms' ability to capture returns from innovation (Teece, 1986; Levin et al., 1987), while bargaining power, switching costs, and firms' positions within platform markets influence how those returns are distributed (Klemperer, 1987; Brandenburger and Stuart, 1996; Rochet and Tirole, 2003). Nordhaus (2004) similarly distinguishes the profits captured by producers from the broader social returns generated by technological innovation.[3] These literatures motivate the paper's separation of economy-wide monetizable value from the cash flow accruing to the firms financing AI infrastructure.

The framework formalizes this distinction through the economy-wide monetizable AI-related value base and the capital-bearing value-capture rate, while the required cash-flow intensity summarizes the investment-side break-even requirement. The economy-wide monetizable AI-related value base includes increases in measured GDP or value added attributable to AI as well as gross monetizable gains accruing to beneficiaries of AI-induced income shifts that do not increase aggregate measured value added.[4] These gross gains are measured on the beneficiary side of the redistribution and can therefore be positive even when gains and losses offset in the aggregate. To avoid double counting, the measure excludes gains already reflected in measured value added. The value base is not a measure of total welfare: nonmonetized consumer surplus, uncompensated time savings, and other nonmarket benefits are excluded unless they generate measured value added or monetizable gains. The capital-bearing value-capture rate relates the AI-attributable unlevered after-tax cash flow available for investment recovery by the five capital-bearing firms to this economy-wide monetizable value base.[5] The

[3] Nordhaus (2004) compares producers' Schumpeterian profits—profits above the normal risk-adjusted return attributable to innovation—with the social returns to technological change. The framework here is conceptually related but not directly comparable: its value base is organized around monetizable value rather than total social returns, its payoff measure is limited to the firms financing the infrastructure, and its payoff concept is AI-attributable unlevered after-tax cash flow available for recovery of the capital expenditure entering the valuation rather than excess innovation profits.

[4] The redistribution component may include gains arising when generative AI displaces existing third-party software or other digital products. Green, Li, and Papanikolaou (2026) distinguish this demand-side product-displacement channel from the labor-cost-saving channel emphasized in conventional AI-exposure measures. When substitution reallocates income across firms without increasing aggregate measured value added, the gross monetizable gain accruing to the beneficiary of that reallocation enters the redistribution component; the corresponding loss does not constitute an additional negative component of the monetizable value base. Any gain already reflected in measured value added is excluded from the redistribution component. Losses arising from the displacement of the capital-bearing firms' own legacy businesses are deducted as cannibalization when cash flow available for investment recovery is calculated.

[5] The numerator of the capital-bearing value-capture rate is the AI-attributable unlevered after-tax cash flow available for recovery of the capital expenditure borne by the firms included in the valuation exercise.

required cash-flow intensity expresses the present value of the capital expenditure entering the valuation relative to the discounted, diffusion-adjusted GDP base over the recovery horizon. Investment recovery therefore depends on whether the economy-wide monetizable value generated by AI, together with the share captured as cash flow by the capital-bearing firms, is sufficient to meet this investment-side requirement. Section 2 provides the formal definitions and derives the break-even condition.

The capital-bearing value-capture rate is not directly observed. The analysis therefore constructs an illustrative calibration using payment and cost evidence available as of mid-2026 while also reporting results over a range of alternative value-capture rates. The calibration uses the share of commercial GenAI merchant-stack payments going to hosting and infrastructure as an illustrative proxy for payments associated with the capital-bearing firms and converts these payments into cash flow using assumptions about costs and taxes. Under the baseline assumptions, this exercise yields a capital-bearing value-capture rate of approximately 5.8 percent. Because the merchant-stack data cover only a subset of the channels through which the five firms may monetize AI-related activity, and the hosting and infrastructure category includes suppliers outside the five firms, this 5.8 percent figure is used as an illustrative calibration value rather than as an estimate of the long-run capital-bearing value-capture rate. Under the investment benchmark, a 20-year recovery horizon, an 8 percent real required return, and the baseline U.S. GDP and diffusion assumptions, the required cash-flow intensity is approximately 1.1 percent. Combining this investment-side requirement with the 5.8 percent calibration value implies that investment recovery requires the long-run level of GDP to be approximately 19 percent above its no-AI path. Using global GDP as the economic base, while holding the investment benchmark and the other calibration inputs fixed, lowers the corresponding requirement to approximately 5 percent.

This paper makes three main contributions. First, it applies the distinction between economy-wide economic gains and private value capture to the quantitative evaluation of AI infrastructure investment. The framework separates the monetizable gains associated with AI across the economy from the portion of those gains that ultimately becomes cash flow available to the firms financing the infrastructure. Second, it develops the required cash-flow intensity as a measure of how large the cash-flow requirement is relative to the economic base available to support investment recovery. Once the investment benchmark, required return, recovery horizon, diffusion path, and economic base are specified, this measure summarizes all of these investment-side inputs in a single break-even requirement. In this sense, it is a sufficient statistic for the investment side of the valuation and can be calculated before specifying either AI's long-run economic effect or the fraction of economy-wide monetizable value that becomes cash flow for the capital-bearing firms. Third, it applies the framework to the 2024–2027 investment benchmark for Amazon, Alphabet, Microsoft, Meta, and Oracle and quantifies how the break-even requirement varies with the required return, recovery horizon, diffusion path, economic base, monetizable income shifts outside measured GDP, the capital-bearing value-capture rate, and post-2027 expansion investment.

The rest of the paper is organized as follows. Section 2 presents the 2024–2027 capital-expenditure benchmark and develops the discounted-cash-flow framework for investment recovery. Section 3 presents the calibration inputs and derives an illustrative value for the capital-bearing value-capture rate using mid-2026 payment and cost evidence. Section 4 reports the valuation results and sensitivity analyses. Section 5 concludes.

2. Investment Benchmark and Discounted-Cash-Flow Framework

This section develops the investment-recovery framework in two parts. Section 2.1 defines the 2024–2027 reported capital-expenditure benchmark and establishes the valuation boundary for Amazon, Alphabet, Microsoft, Meta, and Oracle by specifying which capital expenditures enter the investment benchmark and which corresponding cash flows are counted toward investment recovery. It also explains why the full reported path is used as a gross reported-capex benchmark and, because some of this expenditure may not be attributable to the

AI transition or may have occurred even under the no-AI counterfactual, examines alternative assumptions about the fraction of reported capital expenditure included in the valuation. Section 2.2 then develops the discounted-cash-flow framework under these conventions. It defines the monetizable AI-related value base and the capital-bearing value-capture rate, derives the required cash-flow intensity and the central break-even condition, and relates the financial recovery condition to production-side investment conditions.

2.1. Investment Path and Valuation Boundary

The empirical starting point is the sharp increase in reported and projected capital expenditure by Amazon, Alphabet, Microsoft, Meta, and Oracle, the five firms included in the valuation exercise. Wachter and Wachter (2026) report aggregate capital expenditure of $155 billion in 2022, $150 billion in 2023, $226 billion in 2024, and $381 billion in 2025; they project $755 billion in 2026 and estimate $1.09 trillion in 2027. The 2027 figure is a bottom-up estimate rather than formal management guidance and serves as the final-year input in the 2024–2027 investment path. The 2022–2023 figures provide historical context, while the valuation exercise uses the 2024–2027 portion of the path as the gross reported-capex benchmark.

A strictly incremental NPV test requires the investment and cash-flow sides to be measured relative to the same no-AI counterfactual: only capital expenditure attributable to the AI transition and above the counterfactual investment path should enter the investment side, matched with the corresponding AI-attributable cash flow. The 2024–2027 reported-capex benchmark is broader than this investment concept. Consolidated capital expenditure may include retail logistics, advertising infrastructure, legacy cloud capacity, offices, general network expansion, and other spending that would have occurred even under the no-AI counterfactual. Because public disclosures do not identify the strictly incremental portion, the baseline uses the full reported path as a gross reported-capex benchmark. The sensitivity analysis considers alternative assumptions about the fraction of reported capital expenditure included in the valuation.

The same matching principle applies to the firm boundary: the firms whose capital expenditures enter the valuation must also determine which cash flows count toward investment recovery. Accordingly, the relevant cash-flow concept is the AI-attributable unlevered after-tax cash flow accruing to Amazon, Alphabet, Microsoft, Meta, and Oracle relative to the same no-AI counterfactual and available for recovery of the capital expenditure included in the valuation. Because investment recovery is evaluated at the asset level, independently of how the investment is financed, financing costs are not deducted, consistent with standard asset-side valuation principles (Damodaran, 2024; Koller, Goedhart, and Wessels, 2025). Conceptually, the cash flow available for investment recovery is net of operating costs, cash taxes, working-capital needs, maintenance and replacement investment, and losses from the cannibalization of existing businesses. The illustrative calibration in Section 3 approximates this cash-flow concept using the payment and cost components that can be constructed from the available evidence rather than directly measuring each component of the conceptual definition.

Within this accounting framework, expansion investment is treated separately from the cash-flow measure. The 2024–2027 capital expenditure included in the valuation enters the investment numerator and is therefore not deducted again from cash flow. Post-2027 expansion investment is likewise not deducted from baseline cash flow. In a fully matched extension, its present value would be added to the investment numerator and the cash flows generated by that investment would be included on the cash-flow side. The sensitivity analysis in Table 6 instead isolates the investment-side effect by adding post-2027 expansion investment while holding the maintained value path fixed.

Profits earned by semiconductor suppliers, electricity producers, data-center landlords, or independent model developers may arise from the same AI infrastructure expansion, but they do not contribute to the recovery of capital expenditure borne by the five firms. By contrast, revenue generated through cloud distribution, established relationships with enterprise customers, proprietary data, or AI platforms may contribute to investment recovery even when the underlying value is created elsewhere in the economy, provided that the revenue ultimately increases the cash flow defined above for the same five firms.

Let A denote this set of five firms, and let $I_{f,t}^{\text{reported}}$ denote firm f's reported capital expenditure in year t. Let $\kappa_{f,t} \in [0,1]$ denote the fraction of that expenditure treated as both related to AI infrastructure expansion and incremental relative to a no-AI counterfactual. The aggregate incremental expansion investment entering the present-value comparison is

$$\Delta I_t^A = \sum_{f \in A} \kappa_{f,t} \, I_{f,t}^{\text{reported}}. \qquad (1)$$

The parameter $\kappa_{f,t}$ combines two components that public disclosures do not separately identify. Some reported capital expenditure supports activities outside AI infrastructure, while some AI-related capacity investment would have occurred under the no-AI counterfactual to meet baseline cloud demand. Rather than attempting to estimate these components separately, the valuation exercise uses $\kappa_{f,t}$ as an attribution parameter that determines how much of each firm's reported capital expenditure enters the incremental present-value comparison.

If $\kappa_{f,t}$ is assumed to be constant across firms and years, denote the common value by κ_A. The present value of the investment entering the valuation is then κ_A times the present value of the full reported and projected 2024–2027 capital-expenditure path. The baseline sets $\kappa_A = 1$ and therefore places the entire path assembled by Wachter and Wachter (2026) in the investment numerator as a gross reported-capex benchmark. Values below one represent alternative assumed fractions of reported capital expenditure included in the valuation.

2.2 Discounted-Cash-Flow Framework for Investment Recovery

Building on the investment measure and valuation scope established in Section 2.1, this subsection states the present-value condition for investment recovery. The condition matches the capital expenditure included in the valuation with the corresponding cash flow accruing to the same five firms relative to the no-AI counterfactual and discounts both sides at the required return on AI infrastructure.

The analysis distinguishes economy-wide value creation from the monetization and investment recovery of the capital-bearing firms. AI adoption may create monetizable value across firms and industries, while the capital-bearing firms may generate AI-related revenue through the products, services, platforms, and complementary assets they control. Revenue, however, is not itself the payoff relevant for investment recovery. The relevant payoff is the cash flow accruing to these firms within the valuation scope established in Section 2.1.

Two measures organize the analysis. The required cash-flow intensity summarizes the break-even cash-flow requirement implied by the capital-expenditure benchmark, the diffusion path, the recovery horizon, the GDP normalization, and the required return. The capital-bearing value-capture rate relates the cash flow accruing to the five firms to the economy-wide monetizable value base. Investment is recovered when the resulting cash flow is sufficient, in present value, to match the capital expenditure entering the valuation.

Monetizable AI-Related Value and Capital-Bearing Value Capture

Let S_t^{AI} denote the economy-wide monetizable AI-related value base in year t. The value base measures AI-related value creation and monetizable income shifts across the economy rather than the output or revenue of the capital-bearing firms. It consists of two nonoverlapping components. The first, ΔY_t, is the increase in GDP or measured value added attributable to AI relative to the no-AI counterfactual. It may arise through greater output from a given quantity of real inputs, higher labor productivity, faster software development, improved matching and forecasting, product innovation, and other AI-enabled changes in production. The second, Z_t, captures gross monetizable gains accruing to beneficiaries of AI-induced income shifts that do not increase aggregate measured value added. For example, if AI enables a firm to maintain the same level of output while reducing labor compensation, the resulting increase in profits is a monetizable gain to the firm even though it is offset by a decline in labor income and aggregate measured value added is unchanged. The increase in profits enters Z_t, whereas any increase in measured output or value added generated by the same technology enters ΔY_t. Formally,

$$S_t^{AI} = \Delta Y_t + Z_t. \qquad (2)$$

By construction, Z_t measures the gross gains to beneficiaries of such redistributive shifts rather than the economy-wide net change in income from the redistribution itself. To avoid double counting, it excludes gains already reflected in ΔY_t. Payments between firms for AI services are therefore not added to S_t^{AI} $as\ a$ separate source of value when they represent transfers already recorded in either component. The value base is also narrower than total welfare. Nonmonetized consumer surplus, uncompensated time savings, and other nonmarket benefits are excluded unless they are reflected in measured value added or monetized income shifts.

Let π_t^A denote the cash flow accruing in year t to the firms in A, relative to the no-AI counterfactual, that is available for recovery of the capital expenditure included in the valuation. This cash-flow measure follows the valuation scope established in Section 2.1. Hereafter, unless otherwise stated, cash flow refers to π_t^A. For a recovery horizon T and required return r, define the capital-bearing value-capture rate as the present-value ratio of this cash flow to the monetizable AI-related value base:

$$\theta_A(T) = \frac{\sum_{t=1}^{T}(1+r)^{-t}\pi_t^A}{\sum_{t=1}^{T}(1+r)^{-t}S_t^{AI}}. \qquad (3)$$

The numerator includes only cash flow accruing to the capital-bearing firms, so $\theta_A(T)$ measures their present-value cash flow relative to the economy-wide monetizable AI-related value base rather than an industry or value-chain revenue share.

For each year t, define the period-specific capital-bearing value-capture rate as

$$\theta_{A,t} \equiv \frac{\pi_t^A}{S_t^{AI}}.$$

The horizon-level rate in equation (3) can then be written as the discounted, value-base-weighted average of the period-specific rates:

$$\theta_A(T) = \frac{\sum_{t=1}^{T}(1+r)^{-t}\theta_{A,t}S_t^{AI}}{\sum_{t=1}^{T}(1+r)^{-t}S_t^{AI}}.$$

In the notation, dependence on r is suppressed. When the period-specific rate is assumed to remain constant over the recovery horizon, the time subscript is omitted and both the period-specific and horizon-level rates reduce to the common value θ_A.

The period-specific value-capture rate can be decomposed into a gross-payment ratio and a cash-flow conversion margin. Let P_t^A denote gross AI-related payments received in year t by the firms in A. Define

$$\rho_{A,t} \equiv \frac{P_t^A}{S_t^{AI}} \text{ and } m_{A,t} \equiv \frac{\pi_t^A}{P_t^A}.$$

Then

$$\theta_{A,t} = \frac{\pi_t^A}{S_t^{AI}} = \left(\frac{P_t^A}{S_t^{AI}}\right)\left(\frac{\pi_t^A}{P_t^A}\right) \equiv \rho_{A,t} m_{A,t}. \qquad (4)$$

The first factor, $\rho_{A,t}$, is the gross-payment ratio, which compares the gross AI-related payments received by the capital-bearing firms with the economy-wide monetizable AI-related value base. Because gross payments received by the capital-bearing firms and economy-wide monetizable AI-related value are measured on different accounting bases, $\rho_{A,t}$ is a payment-to-value ratio rather than a share of economy-wide value allocated to these firms. The second factor, $m_{A,t}$, is the cash-flow conversion margin: the portion of those payments that remains as cash flow available for investment recovery. A high payment-to-value ratio therefore need not imply a high capital-bearing value-capture rate if only a small fraction of payments is converted into cash flow. Neither component is directly observed for the capital-bearing firms. Section 3 constructs an illustrative benchmark using merchant-stack payment data and assumptions needed to map those payments into cash flow and the broader monetizable AI-related value base.

Required Cash-Flow Intensity and the Break-Even Condition

Let date 0 denote the beginning of 2024, the valuation date. Capital expenditure in 2024 is dated at 0, while expenditures in 2025–2027 are dated one, two, and three years later. The present value of the capital expenditure entering the valuation is therefore

$$I_0^A(r) \equiv \sum_{\tau=2024}^{2027} (1+r)^{-(\tau-2024)} \Delta I_\tau^A.$$

This timing convention provides a common date for the present-value calculation and does not imply that all capital expenditure occurs at the beginning of each year. For a recovery horizon T, the break-even condition is

$$I_0^A(r) = \sum_{t=1}^{T} (1+r)^{-t} \pi_t^A = \theta_A(T) \sum_{t=1}^{T} (1+r)^{-t} S_t^{AI}. \qquad (5)$$

The first equality requires the present value of the cash flow accruing to the firms in A to equal the present value of the capital expenditure entering the valuation. The second follows from the definition of the capital-bearing value-capture rate.

AI-attributable gains in measured value added are unlikely to materialize immediately. Firms must redesign workflows, accumulate complementary intangible capital, adjust employment and managerial practices, and integrate AI systems into production. To capture this gradual process, the framework assumes that the effective diffusion of AI follows a logistic path:

$$D_t = [1 + \exp\{-k(t - t_0)\}]^{-1}. \quad (6)$$

The diffusion index $D_t \in (0,1)$ governs the timing of AI's effective integration into production. The parameter $k > 0$ determines the speed of diffusion, and t_0 is the date at which the index reaches one-half. The index does not determine the eventual magnitude of the GDP effect. That magnitude is introduced separately through λ:

$$\Delta Y_t = \lambda D_t Y_t^0, \quad (7)$$

where Y_t^0 denotes GDP along the no-AI baseline path and λ is the long-run proportional increase in GDP relative to that path. Accordingly, $\Delta Y_t / Y_t^0 = \lambda D_t$, and the proportional GDP effect approaches λ as D_t approaches one.

Using equation (7), the monetizable AI-related value base is

$$S_t^{AI} = \lambda D_t Y_t^0 + Z_t.$$

For a fixed recovery horizon T, write $\theta_A \equiv \theta_A(T)$. Substituting this expression into equation (5) gives the long-run GDP effect required for break-even:

$$\lambda = \frac{I_0^A(r)/\theta_A - \sum_{t=1}^{T}(1+r)^{-t} Z_t}{\sum_{t=1}^{T}(1+r)^{-t} D_t Y_t^0}. \quad (8)$$

Equation (8) expresses the required long-run GDP effect in terms of the present value of the capital expenditure, the capital-bearing value-capture rate, the diffusion-adjusted GDP base, and the present value of monetizable income shifts.

The baseline quantitative analysis adopts a GDP-only benchmark by setting

$$Z_t = 0 \text{ for all } t,\ S_t^{AI} = \Delta Y_t.$$

Measured GDP provides a common aggregate scale that can be compared directly with existing estimates and scenarios of AI's macroeconomic effects. By contrast, the monetizable income shifts captured by Z_t are not directly observed in national accounts or corporate disclosures. The baseline therefore restricts the monetizable AI-related value base to the measured-value-added component and treats Z_t separately in the sensitivity analysis. As equation (8) shows, a positive present value of Z_t lowers the required long-run GDP effect, holding the capital-bearing value-capture rate, the investment benchmark, the diffusion path, the recovery horizon, and the required return fixed. Equivalently, for a given long-run GDP effect, including Z_t lowers the capital-bearing value-capture rate required for break-even.

Under the GDP-only benchmark, equation (8) reduces to

$$\lambda = \frac{I_0^A(r)}{\theta_A \sum_{t=1}^{T}(1+r)^{-t} D_t Y_t^0}. \quad (9)$$

Equation (9) gives the long-run GDP effect required to recover the capital expenditure included in the valuation under the GDP-only benchmark. The summation term in the denominator is the discounted, diffusion-adjusted GDP base over the recovery horizon. Slower diffusion reduces this base by shifting the realization of AI-attributable GDP gains to later periods and therefore raises the required value of λ, holding the other specified inputs fixed.

Because the annual path of Z_t is not directly observed, suppose that its present value is proportional to the present value of AI-attributable GDP gains:

$$\sum_{t=1}^{T}(1+r)^{-t}Z_t = \zeta \sum_{t=1}^{T}(1+r)^{-t}\Delta Y_t = \zeta\lambda \sum_{t=1}^{T}(1+r)^{-t}D_tY_t^0, \quad \zeta \geq 0.$$

The parameter ζ measures the discounted income-shift component relative to the discounted measured-value-added component. Let $\lambda(\zeta)$ denote the corresponding long-run GDP effect. Combining this proportionality condition with equations (7) and (8) gives

$$\lambda(\zeta) = \frac{I_0^A(r)}{\theta_A(1+\zeta)\sum_{t=1}^{T}(1+r)^{-t}D_tY_t^0} = \frac{\lambda}{1+\zeta}. \qquad (10)$$

Here, λ on the right-hand side is long-run GDP effect under the GDP-only benchmark given by equation (9), so setting $\zeta = 0$ returns that benchmark. Holding the capital-bearing value-capture rate and the other inputs fixed, a larger discounted income-shift component lowers the long-run GDP effect by the factor $1/(1+\zeta)$. For example, $\zeta = 0.25$, 0.5, and 1 reduce the long-run GDP effect to 80 percent, 66.7 percent, and 50 percent of the GDP-only requirement, respectively. Equivalently, for a given long-run GDP effect, monetizable income shifts lower the capital-bearing value-capture rate needed for investment recovery by the same factor.

Equations (9) and (10) show that the long-run GDP effect required for investment recovery depends on the size of the investment benchmark relative to the GDP base available to generate AI-related gains over the recovery horizon. Define the discounted, diffusion-adjusted GDP base as $B_T(r)$ and the required cash-flow intensity as $H_T(r)$:

$$B_T(r) \equiv \sum_{t=1}^{T}(1+r)^{-t}D_tY_t^0 \quad H_T(r) \equiv \frac{I_0^A(r)}{B_T(r)}. \qquad (11)$$

The required cash-flow intensity therefore measures the present value of the capital expenditure included in the valuation relative to the discounted, diffusion-adjusted GDP base over the recovery horizon. Using equation (5), equation (11) can equivalently be written as

$$I_0^A(r) = H_T(r)\sum_{t=1}^{T}(1+r)^{-t}D_tY_t^0 = \sum_{t=1}^{T}(1+r)^{-t}[H_T(r)D_tY_t^0].$$

This identity provides a convenient interpretation of the required cash-flow intensity. If the required cash-flow path is assumed to be proportional to the diffusion-adjusted GDP path, $D_tY_t^0$, then $H_T(r)$ is the constant coefficient that makes the present value of that cash-flow stream equal to $I_0^A(r)$. Under this proportional-path interpretation, the required cash flow as a share of baseline GDP in year t is $H_T(r)D_t$. Under a proportional annual cash-flow path, $H_T(r)$ is the constant coefficient that equates the present value of cash flow with the investment benchmark. Because investment recovery is evaluated in present-value terms, cash flow need not follow a particular year-by-year path. Different annual cash-flow patterns can satisfy the break-even condition as long as their discounted present value is sufficient to recover the investment.

The required cash-flow intensity, $H_T(r)$, summarizes the investment side of the break-even condition in a single statistic. It measures the present-value cash flow required for investment recovery relative to the discounted, diffusion-adjusted GDP base. Its value reflects the size and timing of the capital expenditure included in the valuation, the required return, the recovery horizon, the diffusion path, and the GDP base used in the calibration. Because neither the capital-bearing value-capture rate nor the income-shift parameter ζ enters its definition, $H_T(r)$ can be calculated independently of assumptions about value capture and monetizable income shifts. Conditional on these inputs, $H_T(r)$ is a sufficient statistic for the investment side of the break-even condition.

Substituting equation (11) into equation (9) gives the central break-even condition under the GDP-only benchmark:

$$\theta_A \lambda = H_T(r). \qquad (12)$$

Equation (12) has a direct economic interpretation. The long-run GDP effect, λ, determines the scale of AI-attributable GDP gains, while the capital-bearing value-capture rate, θ_A, determines how much of those gains reaches the capital-bearing firms as cash flow. Their product, $\theta_A \lambda$, therefore measures the cash flow generated for investment recovery relative to the discounted, diffusion-adjusted GDP base. Break-even requires this cash-flow intensity to equal the investment-side requirement, $H_T(r)$. Under the income-shift sensitivity in equation (10), the corresponding condition is

$$\theta_A(1+\zeta)\lambda(\zeta) = H_T(r).$$

More generally, under the GDP-only benchmark, investment recovery requires

$$\theta_A \lambda \geq H_T(r),$$

with equality at break-even. The quantitative analysis therefore proceeds in three steps. First, it calculates $H_T(r)$ from the capital expenditure included in the valuation, the required return, the recovery horizon, the diffusion path, and the GDP base used in the calibration. Second, it uses payment and cost evidence to construct an illustrative calibration value for θ_A and considers alternative long-run values of the capital-bearing value-capture rate. Third, it combines $H_T(r)$ with alternative values of θ_A and, in the sensitivity analysis, ζ, to determine the corresponding long-run GDP effect.

Relation to Production-Side Investment Conditions

The discounted-cash-flow framework developed above can be related to production-side conditions for AI investment. Production-side conditions evaluate whether AI capital generates sufficient economy-wide output relative to its cost. For the investment-recovery question studied here, an additional consideration is how much of the value generated by AI accrues as cash flow to the firms financing the investment. This distinction is familiar from the innovation and appropriability literatures, which emphasize that firms may capture only part of the value created by an innovation when benefits accrue to users, consumers, imitators, or owners of complementary assets (Teece, 1986; Levin et al., 1987; Nordhaus, 2004). The distinction is particularly relevant for general-purpose technologies, whose economic effects depend on complementary innovation and adoption across upstream and downstream sectors (Bresnahan and Trajtenberg, 1995).

To formalize the production-side investment criterion, let K_τ denote AI capital installed at date τ, and let

$$\mathcal{K}_t \equiv \{K_\tau\}_{\tau=0}^{t}$$

denote the collection of AI-capital vintages available for production at date t. Let $Y_t\ (\mathcal{K}_t)$ denote output at date t. With output and investment measured in a common numeraire and the price of installed capital normalized to one, a generic production-side net present value at date 0 is

$$V_0 = \sum_{t=0}^{T} Q_{0,t}\left[Y_t\ (\mathcal{K}_t) - I_t^a\right]. \qquad (13)$$

Here, I_t denotes investment expenditure, and $Q_{0,t}$ is the date-0 value of one unit of output delivered at date t.[6] Suppose that one unit of investment expenditure at date t installs one unit of K_t, so that the marginal installation cost is one. Consistent with the cash-flow timing used above, suppose that capital installed at date t begins contributing to output from date $t+1$. The survival, depreciation, and output effects of that capital are reflected in its marginal contributions to output from date $t+1$ and onward. Abstracting from adjustment costs, the first-order condition for an interior investment choice is

$$Q_{0,t} = \sum_{s=t+1}^{T} Q_{0,s}\frac{\partial Y_s\ (\mathcal{K}_s)}{\partial K_t}. \qquad (14)$$

The left-hand side is the date-0 value of the unit installation cost, while the right-hand side is the date-0 value of the marginal output generated over the remaining life of the capital. Equation (14) therefore requires the discounted marginal output from an additional unit of AI capital to equal its installation cost. Equation (14) values total marginal output, leaving the allocation of that value between the capital-bearing firms and other economic agents unspecified.

To connect the production-side condition to investment recovery by the capital-bearing firms, suppose that the capital-bearing value-capture rate is constant over the recovery horizon. Cash flow in year s is then

$$\pi_s^A = \theta_A S_s^{AI}.$$

An additional unit of AI capital installed at date t can contribute to monetizable AI-related value, and hence to cash flow, in subsequent years. With the unit installation cost normalized to one and holding θ_A fixed, the marginal break-even condition for an additional unit of K_t is

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[6] The pricing notation is deterministic to maintain consistency with the calibration. Under the constant-rate specification, $Q_{0,t} = (1+r)^{-t}$. Under uncertainty, future marginal payoffs would instead be valued using conditional expectations and the relevant stochastic discount factor.

$$Q_{0,t} = \sum_{s=t+1}^{T} Q_{0,s} \frac{\partial \pi_s^A}{\partial K_t} = \theta_A \sum_{s=t+1}^{T} Q_{0,s} \frac{\partial S_s^{AI}}{\partial K_t}. \quad (15)$$

Equation (15) incorporates the value-capture margin relevant for private investment recovery. An additional unit of AI capital may generate monetizable value that does not fully accrue to the capital-bearing firms; only the fraction θ_A that reaches these firms as cash flow is available for recovery of their investment. Marginal break-even occurs when the present value of this cash flow equals the installation cost of the additional unit of capital. Under additional homogeneity assumptions, this marginal condition aggregates to the break-even condition in equation (12).[7] Equation (14) evaluates the full marginal output generated by AI capital, whereas equation (15) evaluates the cash-flow payoff accruing to the firms financing the investment. The two conditions therefore need not coincide when $\theta_A < 1$.

Under the GDP-only benchmark, the marginal AI-attributable value in equation (15) corresponds to the marginal output gain considered in equation (14). The two conditions differ in the payoff relevant to the investment decision. Equation (14) evaluates the economy-wide marginal output generated by an additional unit of AI capital, whereas equation (15) evaluates the cash flow from that value that accrues to the capital-bearing firms. When $\theta_A = 1$, the two payoffs coincide. When $\theta_A < 1$, part of the value generated by AI capital accrues to other firms, workers, suppliers, or consumers, so the cash flow available to the capital-bearing firms is smaller than the economy-wide value created. This difference is the familiar appropriability margin represented here by the capital-bearing value-capture rate. Equation (15) applies this margin to the recovery of an additional unit of AI capital, whereas equation (12) evaluates recovery of the full capital expenditure included in the valuation.

3. Calibration Inputs and an Illustrative Value-Capture Benchmark

This section specifies the investment, macroeconomic, and payment-and-cost inputs used in the quantitative analysis and constructs an illustrative value-capture calibration using evidence available as of mid-2026. Following equation (4), the calibration decomposes the capital-bearing value-capture rate, θ_A, into the gross-payment ratio, ρ_A, and the cash-flow conversion margin, m_A, so that $\theta_A = \rho_A m_A$. For the calibration, these components are held constant over the recovery horizon, and their time subscripts are therefore suppressed. Merchant-stack payment data, together with the mapping from recorded payments to the broader monetizable AI-related value base, inform ρ_A. Azhar et al.'s (2026) estimate of the share of infrastructure revenue remaining after depreciation, together with assumptions about other cash operating costs and cash taxes, informs m_A. Because θ_A is defined over the full recovery horizon, evidence from mid-2026 does not identify its long-run value. Under the baseline assumptions, however, the calculation yields an illustrative calibration value for θ_A of approximately 5.8 percent. The valuation analysis combines this calibration value and alternative assumed values of θ_A with the required cash-flow intensity, $H_T(r)$, to calculate the corresponding required long-run GDP effect, λ.

[7] Under unit installation costs, a constant capital-bearing value-capture rate, and linear homogeneity of S_s^{AI} in the installed AI-capital vintages, multiplying equation (15) by K_t, summing over vintages, and applying Euler's theorem gives $\sum_t Q_{0,t} K_t = \theta_A \sum_s Q_{0,s} S_s^{AI}$. With the same timing and discounting conventions and the GDP-only benchmark, this condition reduces to equation (12).

All dollar amounts are expressed in trillions of constant 2024 dollars unless otherwise noted. The beginning of 2024 is the valuation date, $t = 0$. Capital expenditure reported for 2024 is dated at $t = 0$, while the 2025–2027 amounts are dated at $t = 1, 2,$ and 3, respectively, and discounted to $t = 0$ at the required return, r. Given the short 2024–2027 interval, the reported and projected nominal capital-expenditure amounts are treated as constant-2024-dollar inputs.

The capital-expenditure inputs follow Wachter and Wachter (2026). Table 1 reports the 2024–2027 capital-expenditure path and its present value at real required returns of $r = 8\%$ and $r = 15\%$. The undiscounted total is \$2.45 trillion; the corresponding present values are \$2.09 trillion at $r = 8\%$ and \$1.85 trillion at $r = 15\%$. The baseline uses $r = 8\%$, while the $r = 15\%$ case examines sensitivity to a higher real required return. The baseline sets $\kappa_A = 1$, so the full reported capital-expenditure path enters the investment numerator. Lower values of κ_A are considered in the sensitivity analysis. Because κ_A scales the investment numerator proportionally, holding the discounted, diffusion-adjusted GDP base and the capital-bearing value-capture rate, θ_A, fixed, the present value of investment, the required cash-flow intensity, $H_T(r)$, and the required long-run GDP effect, λ, are all proportional to κ_A. At $r = 8\%$, setting $\kappa_A = 0.75$ lowers the required long-run GDP effect from 11.1 percent to 8.3 percent when $\theta_A = 10\%$, and from 22.1 percent to 16.6 percent when $\theta_A = 5\%$. Setting $\kappa_A = 0.50$ lowers the corresponding requirements to 5.5 percent and 11.1 percent.

The baseline constructs the discounted, diffusion-adjusted GDP base using the U.S. no-AI GDP path, with 2024 U.S. GDP level set at \$29.2 trillion (World Bank, 2026; Wachter and Wachter, 2026). The no-AI GDP path is assumed to grow at a constant annual real rate g, so that $Y_t^0 = Y_0^0(1 + g)^t$ with g=0.02 in the baseline calibration. Because the five capital-bearing firms in A operate globally, the analysis also reports results using global GDP as the economic base, while holding the investment benchmark and the other calibration inputs fixed. The baseline diffusion path sets $k = 0.5$ and $t_0 = 7$ years. These values specify a gradual adoption path consistent with the general-purpose-technology literature, which emphasizes that complementary investment, organizational restructuring, learning, and heterogeneous adoption across firms and sectors can delay the effects of new technologies on aggregate production (Bresnahan and Trajtenberg, 1995; Helpman and Trajtenberg, 1996; Brynjolfsson, Rock, and Syverson, 2019, 2021). With $t = 0$ corresponding to 2024, the diffusion index is approximately 3 percent in 2024, 12 percent in 2027, 27 percent in 2029, 50 percent in 2031, and 82 percent in 2034. The increase from 10 percent to 90 percent takes approximately nine years. The calibration therefore allows for a substantial lag between the availability of AI technologies and their effective integration into economy-wide production.

The calculation proceeds in two steps. First, data on commercial payments across the GenAI merchant stack are used to construct the gross-payment ratio, ρ_A, by relating payments associated with the capital-bearing firms to the broader monetizable AI-related value base. Second, these payments are converted into cash flow available for investment recovery using the reported share of infrastructure revenue remaining after depreciation, together with assumptions about other cash operating costs and the cash tax rate.

The first stage uses Azhar et al.'s (2026) estimates of the distribution of commercial payments across applications, models, and hosting and infrastructure. Let M_t denote total commercial payments after removing double counting across these market layers, and let M_t^H denote payments in the hosting and infrastructure category. Azhar et al. (2026, p. 55) report that this category accounted for approximately 82 percent of total payments in the first quarter of 2026. Because the category includes infrastructure suppliers outside the five capital-bearing firms in A, the observed share is not a direct measure of the payments received by those firms. The benchmark therefore uses the hosting and infrastructure share as an illustrative proxy for their share of recorded commercial payments. Define

$$\omega_{A,t} = \frac{M_t^H}{M_t}. \qquad (16)$$

The proxy may overstate the share of recorded payments accruing to the firms in A because the hosting and infrastructure category includes other suppliers. At the same time, the payment data may omit AI-related monetization embedded in the firms' existing businesses, so the overall direction of measurement bias is not determined. The benchmark sets $\omega_A = 0.82$, using the first-quarter 2026 value throughout the recovery horizon.

The observed hosting and infrastructure share is defined relative to total recorded commercial payments and is therefore not itself a share of the broader monetizable AI-related value base, S_t^{AI}. To relate the two measurement bases, define

$$\mu_t \equiv \frac{S_t^{AI}}{M_t}. \qquad (17)$$

The parameter μ_t measures the broader monetizable AI-related value base relative to recorded commercial payments. Values above one indicate that monetizable value exceeds recorded commercial payments, whereas values below one indicate that recorded payments exceed contemporaneously realized monetizable value. The gross-payment ratio defined in Section 2 is therefore

$$\rho_{A,t} = \frac{\omega_{A,t} M_t}{S_t^{AI}} = \frac{\omega_{A,t}}{\mu_t}. \qquad (18)$$

Because the available data do not directly identify the relationship between recorded commercial payments and the broader monetizable AI-related value base, the baseline sets $\mu = 1$. Suppressing time subscripts, this implies $\rho_A = \omega_A = 0.82$.

The second stage constructs an illustrative cash-flow conversion margin, m_A, using the share of infrastructure revenue remaining after depreciation together with assumptions about other cash operating costs and cash taxes. In the calibration, m_A represents the fraction of gross payments received by the firms in A that remains available for investment recovery. Azhar et al. (2026, p. 34) report that, in the first quarter of 2026, GenAI infrastructure revenue net of associated depreciation was approximately 19 percent of revenue for hyperscalers and neocloud providers. Let R^H and Dep^H denote revenue and depreciation for this infrastructure category. Thus,

$$\frac{R^H - \mathrm{Dep}^H}{R^H} = 0.19. \qquad (19)$$

Because this category includes firms outside A, the 19 percent ratio is used as an illustrative proxy for the corresponding post-depreciation revenue ratio of the capital-bearing firms. The depreciation deduction is treated as a proxy for the maintenance and replacement investment required to sustain the installed infrastructure base. Let C^H denote cash operating costs other than depreciation, and let τ denote the cash tax rate. The illustrative cash-flow conversion margin is then

$$m_A = \left(0.19 - \frac{C^H}{R^H}\right)(1 - \tau). \qquad (20)$$

Equation (20) deducts other cash operating costs from the post-depreciation revenue ratio and applies the cash tax rate to the resulting pretax margin.

The operating-cost calibration is informed by Michael and Cottier's (2026) stylized Epoch AI model of a one-gigawatt data center owned and operated by a U.S. hyperscaler. The model reports approximately \$38 billion in upfront capital expenditure, \$0.9 billion in annual cash operating expenses, and \$8.5 billion in annualized total cost of ownership. Because the model does not report revenue, it does not directly identify the cash operating-cost ratio C^H/R^H. Let R^{1GW} and C^{1GW} denote annual revenue and cash operating costs for the stylized facility. If a mature facility generates sufficient revenue to cover its annualized total cost, then $R^{1GW} \geq$ \$8.5 billion. Given $C^{1GW} = \$0.9$ billion, this condition implies

$$\frac{C^{1GW}}{R^{1GW}} \leq \frac{0.9}{8.5} \approx 10.6\%. \qquad (21)$$

The baseline adopts $C^H/R^H = 0.10$ as an illustrative operating-cost ratio consistent with this condition. The calibration also considers alternative cash operating-cost ratios of $C^H/R^H = 0.075$ and 0.15. These values bracket the baseline ratio of 0.10 and allow for variation in operating costs, revenue, or both.

With the baseline cash operating-cost ratio, $C^H/R^H = 0.10$, and cash tax rate, $\tau = 0.21$, equation (20) gives

$$m_A = (0.19 - 0.10)(1 - 0.21) \approx 0.071$$

To translate this cash-flow conversion margin into the capital-bearing value-capture rate, consider first the baseline mapping $\mu = 1$. Equation (18) then gives $\rho_A = \omega_A = 0.82$, so that

$$\theta_A = \rho_A m_A = \omega_A m_A = 0.82 \times 0.071 \approx 0.058,$$

or approximately 5.8 percent. The alternative cash operating-cost ratios of $C^H/R^H = 0.075$ and 0.15 imply cash-flow conversion margins of approximately 9.1 percent and 3.2 percent, respectively. At $\mu = 1$, these correspond to capital-bearing value-capture rates of approximately 7.4 percent and 2.6 percent. More generally, combining equations (18) and (20), the illustrative calibration value for the capital-bearing value-capture rate varies with μ according to

$$\theta_A(\mu) = \frac{\omega_A m_A}{\mu} \approx \frac{0.058}{\mu}. \qquad (22)$$

Holding the baseline operating-cost and tax assumptions fixed, the corresponding calibration values are approximately 5.8 percent at $\mu = 1$, 2.9 percent at $\mu = 2$, and 1.9 percent at $\mu = 3$.

Table 2 summarizes the calibration inputs used to calculate the required cash-flow intensity, $H_T(r)$, and to derive calibration values for the capital-bearing value-capture rate, θ_A. The investment, GDP, required-return, recovery-horizon, and diffusion assumptions determine $H_T(r)$. The payment-share proxy, ω_A, the monetizable-value-to-payment ratio, μ, and the cash-flow conversion margin, m_A, determine the calibration value for θ_A. The table also reports selected alternative values for parameters varied in the subsequent sensitivity analysis.

Table 3 summarizes the illustrative value-capture calibration values derived from the mid-2026 payment and cost evidence and reports their sensitivity to the cash operating-cost ratio, the cash tax rate, and the monetizable-value-to-payment ratio. Under the baseline assumptions $\omega_A = 0.82$, $C^H/R^H = 0.10$, $\tau = 0.21$, and $\mu = 1$, the cash-flow conversion margin is approximately 7.1 percent, yielding a capital-bearing value-capture rate of approximately 5.8 percent. The reduction from the 82 percent payment-share proxy to the 5.8 percent value-capture rate reflects the conversion of gross payments into cash flow after depreciation, other cash operating costs, and cash taxes. Varying the cash operating-cost ratio from 7.5 to 10 and 15 percent yields value-capture rates of approximately 7.4, 5.8, and 2.6 percent, respectively. Holding the baseline operating-cost ratio fixed, reducing the cash tax rate from 21 percent to 15 percent raises the value-capture rate from approximately 5.8 percent to 6.3 percent. Increasing μ reduces the gross-payment ratio, $\rho_A = \omega_A/\mu$, and therefore the value-capture rate. At $\mu = 2$, $\rho_A = 0.41$ and θ_A is approximately 2.9 percent; at $\mu = 3$, ρ_A is approximately 27.3 percent and θ_A is approximately 1.9 percent. Over the ranges reported in the table, the calibration value is more sensitive to the cash operating-cost ratio and μ than to the cash tax rate.

4. Valuation Implications of Alternative Value-Capture Rates

This section combines the required cash-flow intensity with alternative values of the horizon-level capital-bearing value-capture rate, θ_A, to evaluate the long-run GDP effects required for investment recovery. Under the GDP-only benchmark, equation (12) implies

$$\lambda = \frac{H_T(r)}{\theta_A}$$

so the required long-run GDP effect is inversely related to the capital-bearing value-capture rate. Because $H_T(r)$ is determined by the investment benchmark, the required return, the recovery horizon, the diffusion path, and the GDP base independently of θ_A, varying θ_A isolates the effect of value capture on the break-even GDP requirement.

Baseline Valuation Results

The baseline specification uses the full reported 2024–2027 capital-expenditure path as the investment benchmark, U.S. GDP as the economic base, a 20-year recovery horizon, the baseline logistic diffusion path, and a terminal value of zero. At the baseline real required return of 8 percent, the calculation is

$$I_0^A(0.08) = \$2.091 \text{ trillion}, \; B_{20}(0.08) = \$189.2 \text{ trillion},$$
$$H_{20}(0.08) = \frac{I_0^A(0.08)}{B_{20}(0.08)} = \frac{2.091}{189.2} = 1.105\%, \; \lambda = \frac{H_{20}(0.08)}{\theta_A} = \frac{1.105\%}{5.8\%} = 19.1\%.$$

Thus, at the 5.8 percent calibration value for θ_A, recovery of the full reported 2024–2027 capital-expenditure benchmark requires a long-run GDP effect of approximately 19 percent. At the alternative real required return of 15 percent, holding the other baseline inputs fixed, the required cash-flow intensity is approximately 2.00 percent. Appendix A reports the underlying calculations and the corresponding infinite-horizon result. Table 4 combines the required cash-flow intensities with alternative values of θ_A to derive the associated required long-run GDP effects.

Using U.S. GDP as the economic base, Table 4 reports required long-run GDP effects of 22.1 percent at $r = 8\%$ and 40.0 percent at $r = 15\%$ when $\theta_A = 5\%$. Using instead the 5.8 percent calibration from Section 3 lowers the corresponding requirements to approximately 19.0 percent and 34.5 percent, with all other calibration inputs held fixed. Table 4 includes the 5 percent case in a grid of alternative value-capture rates, while the result based on the 5.8 percent calibration value is reported separately. Using global rather than U.S. GDP as the economic base further lowers the required long-run GDP effect because the same investment benchmark is evaluated against a larger discounted, diffusion-adjusted GDP base. With 2024 global GDP of approximately \$111.0 trillion, the required effect at $r = 8\%$ and $\theta_A = 5\%$ falls from 22.1 percent under the U.S. GDP base to 5.8 percent under the global GDP base.

Using U.S. GDP as the economic base and a real required return of 8%, the required long-run GDP effect rises from 5.5 percent at $\theta_A = 20\%$ to 11.1 percent at $\theta_A = 10\%$, 22.1 percent at $\theta_A = 5\%$, and 110.5 percent at $\theta_A = 1\%$. Under the full 2024–2027 reported capital-expenditure benchmark and a 20-year recovery horizon, the results show that the GDP effect required for investment recovery increases sharply as the capital-bearing value-capture rate declines. This inverse relationship follows directly from the break-even condition, $\lambda =$

$H_T(r)/\theta_A$; when a smaller fraction of economy-wide AI-related value reaches the capital-bearing firms as cash flow, a larger aggregate economic payoff is required to recover the same investment benchmark.

The results above use the GDP-only benchmark, which sets $\zeta = 0$. A separate sensitivity allows part of the monetizable AI-related value base to arise from income shifts outside measured GDP. Under the proportional present-value parameterization introduced in Section 2, holding the horizon-level capital-bearing value-capture rate, θ_A, fixed, the required long-run GDP effect is multiplied by $1/(1+\zeta)$. For example, at $\theta_A = 5\%$ and $r = 8\%$, the required long-run GDP effect using U.S. GDP as the economic base falls from 22.1 percent to 17.7 percent when $\zeta = 0.25$, 14.7 percent when $\zeta = 0.5$, and 11.1 percent when $\zeta = 1$. Using global GDP as the economic base, the corresponding requirement falls from 5.8 percent to 4.6 percent, 3.9 percent, and 2.9 percent, respectively.

For perspective, the required GDP effects can be compared in magnitude with the scenarios in Wachter and Wachter (2026). Their moderate, transformative, and singularity cases generate approximately 5, 20, and 58 percentage points of additional cumulative GDP growth by 2030, respectively. In the transformative case, an additional productivity boom during 2028–2030 raises AI-sector productivity by a factor of approximately 2.7. Using U.S. GDP as the economic base, a real required return of 8 percent, and the 5.8 percent calibration value from Section 3, the baseline calculation yields a required long-run GDP effect of approximately 19 percent, close in numerical magnitude to their transformative outcome. The two figures describe different objects: the present analysis reports a proportional increase in the long-run level of GDP over a 20-year recovery horizon, whereas Wachter and Wachter report additional cumulative GDP growth through 2030.

Timing and Horizon Sensitivity

Timing matters because the investment is concentrated in 2024–2027, whereas diffusion and the associated value creation unfold gradually. Table 5 reports the sensitivity of the required long-run GDP effect to the recovery horizon, holding the 2024–2027 investment benchmark fixed and assuming no additional expansion investment after 2027. Using U.S. GDP as the economic base and $r = 8\%$, shortening the horizon from 20 to 10 years raises the required long-run GDP effect from 22.1 percent to 58.3 percent at $\theta_A = 5\%$, while extending the horizon to 50 years lowers it to 13.1 percent. A shorter recovery horizon requires a larger GDP effect because less time is available for the associated cash flows to recover the investment benchmark. Conversely, extending the horizon lowers the required GDP effect. The 50- and 100-year results approach the infinite-horizon values reported in Appendix Table A1 because the present-value contribution of increasingly distant cash flows becomes small at an 8 percent real required return.

The required long-run GDP effect is also sensitive to the timing of diffusion. Using U.S. GDP as the economic base, $r = 8\%$,, a 20-year recovery horizon, and $\theta_A = 5\%$, moving the diffusion midpoint from the baseline value of $t_0 = 7$ years to $t_0 = 5$ years lowers the required value of λ from 22.1 percent to 18.4 percent. Delaying the midpoint to 10 and 12 years raises the requirement to 30.7 percent and 40.0 percent, respectively. By contrast, changing the logistic slope from the baseline value of $k = 0.5$ to $k = 0.3$ or $k = 0.8$, while holding $t_0 = 7$, changes the required GDP effect only slightly, to 22.0 percent and 22.4 percent. In this calibration, the timing of the diffusion midpoint therefore matters substantially more than the slope of the diffusion path.

Post-2027 Investment

The baseline valuation assumes no post-2027 expansion investment and defines the cash-flow measure net of maintenance and replacement investment. Appendix B first shows that an equivalent formulation can instead measure cash flow before these outlays and add maintenance and replacement investment to the investment requirement. When the cash-flow and investment boundaries are defined consistently, the two formulations yield the same break-even condition. This accounting equivalence clarifies why maintenance and replacement investment already deducted from the baseline cash-flow measure should not be added again to the investment numerator. Appendix B then extends the framework to post-2027 expansion investment, which lies outside the 2024–2027 reported capital-expenditure benchmark. Table 6 applies this extension while retaining the baseline cash-flow definition. It sets the present value of post-2027 expansion investment equal to 25, 50, or 100 percent of the $2.091 trillion present value of the 2024–2027 benchmark. The additional investment is added to the investment numerator while the discounted, diffusion-adjusted GDP base and the capital-bearing value-capture rate are held fixed. The calculation does not specify a separate additional value path associated with the post-2027 investment and therefore isolates the effect of the larger investment requirement.

Table 6 shows that the investment-recovery requirement rises in proportion to the present value of additional investment under these assumptions. With no post-2027 expansion investment, the required cash-flow intensity is 1.11 percent and the required long-run GDP effect is 22.1 percent at $\theta_A = 5\%$. When the present value of additional investment equals 25, 50, and 100 percent of the 2024–2027 benchmark, the required cash-flow intensity rises to 1.38, 1.66, and 2.21 percent, respectively, and the corresponding required long-run GDP effects rise to 27.6, 33.2, and 44.2 percent. Because the discounted, diffusion-adjusted GDP base and θ_A are held fixed, these increases reflect only the larger investment amount that must be recovered.

5. Conclusion

This paper evaluates the conditions under which the reported 2024–2027 AI infrastructure investment of Amazon, Alphabet, Microsoft, Meta, and Oracle can earn its required return. The discounted-cash-flow framework separates the cash-flow requirement implied by the investment benchmark from economy-wide monetizable AI-related value and from the cash flow accruing to the capital-bearing firms. The required cash-flow intensity summarizes the investment-side requirement independently of assumptions about AI's long-run economic effect and the capital-bearing value-capture rate, allowing investment recovery to be evaluated without imposing a particular productivity forecast or point estimate of long-run value capture.

In the baseline 20-year U.S. calibration, the full reported 2024–2027 capital-expenditure benchmark implies a required cash-flow intensity of approximately 1.1 percent. At the 5.8 percent calibration value for the capital-bearing value-capture rate, investment recovery under the GDP-only specification requires the long-run level of GDP to be approximately 19 percent above its no-AI path. The required GDP effect is higher when the capital-bearing value-capture rate is lower, diffusion is delayed, the required return is higher, the recovery horizon is shorter, or additional post-2027 expansion investment is included. It is lower when a smaller fraction of reported capital expenditure is included in the valuation, a larger GDP base is used, or monetizable income shifts outside measured GDP are included.

For investment recovery, economy-wide productivity gains and AI-related revenue growth matter only to the extent that they generate cash flow available for recovery of the investment by the capital-bearing firms. AI may create substantial value for user firms, workers, consumers, and suppliers while generating insufficient cash flow for the firms financing the infrastructure to recover their investment. A higher capital-bearing value-capture rate lowers the economy-wide payoff required for recovery. The framework therefore identifies the combinations of economy-wide value creation and firm-level value capture consistent with recovery of the reported investment benchmark. If the cash flow accruing to the capital-bearing firms is insufficient to recover the benchmark at the required return, the investment expansion may prove excessive ex post even if AI creates substantial value elsewhere in the economy.

The analysis is a quantitative valuation exercise rather than a structural model of AI production, diffusion, investment, and value capture. A structural extension would jointly model firms' investment decisions, the accumulation and retirement of successive capital vintages, AI diffusion across firms and sectors, and the evolution of prices, costs, competition, and value capture. Such an analysis would require more detailed firm- and sector-level evidence linking AI-related investment and adoption to revenue, costs, and cash flow, which lies beyond the scope of this paper.

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Table 1. Reported investment path and present value

Year	Reported capital expenditure	Present value at $r = 8\%$	Present value at $r = 15\%$
2024	$0.226T	$0.226T	$0.226T
2025	$0.381T	$0.353T	$0.331T
2026	$0.755T	$0.647T	$0.571T
2027	$1.090T	$0.865T	$0.717T
Total	$2.452T	$2.091T	$1.845T

Note: Reported capital-expenditure amounts are treated as constant-2024-dollar inputs. The 2024 amount is dated at $t = 0$, and the 2025–2027 amounts are discounted to the beginning of 2024. Dollar amounts are in trillions (T).

Source: Wachter and Wachter (2026); author's calculations.

Table 2. Calibration Inputs and Selected Sensitivity Values

Input or parameter	Baseline value	Interpretation
PV investment, I_0^A, at $r = 8\%$	\$2.091T	Present value of the full 2024–2027 reported capital-expenditure path under $\kappa_A = 1$
r	8% benchmark; 15% stress	Real required return used to discount investment and cash-flow paths
Y_0^0	\$29.2T	2024 U.S. GDP normalization used to initialize the no-AI path
g	2%	Annual real growth rate of the no-AI GDP path
$k,\ t_0$	0.5; 7 years	Speed and midpoint of the logistic diffusion path
κ_A	1.0	Assumed fraction of the reported capital-expenditure path included in the investment numerator; $\kappa_A = 1$ defines the full gross reported-capex benchmark.
ω_A	0.82	Illustrative proxy based on the reported hosting and infrastructure category share; the source category is broader than A.
μ	1.0 benchmark; >1 in sensitivity	Ratio of the monetizable AI-related value base to recorded commercial payments
$(R^H - \mathrm{Dep}^H)/R^H$	0.19	Share of infrastructure revenue remaining after depreciation expense but before other operating costs
C^H/R^H	10% central; 15% high-cost	Cash operating costs excluding depreciation and capital-recovery charges, as a share of infrastructure revenue
τ	21% baseline; 15% sensitivity	Illustrative statutory-rate proxy for the cash tax rate used in constructing the cash-flow conversion margin

Note: Baseline values are used in the main 20-year U.S. calibration. Sensitivity values are varied as specified in the text and subsequent tables. Dollar amounts are expressed in trillions of constant 2024 dollars unless otherwise noted.

Source: Wachter and Wachter (2026); Azhar et al. (2026); Michael and Cottier (2026); World Bank (2026); Internal Revenue Service (2026); author's calculations.

Table 3. Illustrative Value-Capture Calibration and Sensitivity

Case	μ	ρ_A	C^H/R^H	τ	$\boldsymbol{m_A}$	$\boldsymbol{\theta_A}$
Lower operating-cost ratio	1.0	82.0%	7.5%	21%	9.1%	7.4%
Baseline	1.0	82.0%	10%	21%	7.1%	5.8%
Higher operating-cost ratio	1.0	82.0%	15%	21%	3.2%	2.6%
Lower cash-tax rate	1.0	82.0%	10%	15%	7.7%	6.3%
μ = 2	2.0	41.0%	10%	21%	7.1%	2.9%
μ = 3	3.0	27.3%	10%	21%	7.1%	1.9%

Note: All cases hold the illustrative proxy $\omega_A$at 82 percent and the post-depreciation infrastructure revenue margin at 19 percent. The 82 percent value is based on the hosting and infrastructure category reported by Azhar et al. (2026), which is broader than the five firms in A. The gross-payment ratio, ρ_A, adjusts this proxy for the monetizable-value-to-payment ratio, μ. The cash-flow conversion margin, m_A, starts from the post-depreciation infrastructure revenue margin, deducts additional cash operating costs other than depreciation and economic capital-recovery charges, and applies the cash tax rate. The capital-bearing value-capture calibration value, θ_A, combines the gross-payment ratio and the cash-flow conversion margin. Values are rounded to one decimal place.
Source: Azhar et al. (2026); Michael and Cottier (2026); Internal Revenue Service (2026); author’s calculations.

Table 4. Required Long-Run GDP Effects under Alternative Capital-Bearing Value-Capture Rates

Capital-bearing value-capture rate, θ_A	U.S. GDP base, r = 8%	Global GDP base, r = 8%	U.S. GDP base, r = 15%	Global GDP base, r = 15%
20%	5.5%	1.5%	10.0%	2.6%
10%	11.1%	2.9%	20.0%	5.3%
5%	22.1%	5.8%	40.0%	10.5%
2.5%	44.2%	11.6%	80.0%	21.0%
1%	110.5%	29.1%	200.0%	52.6%

Note: The U.S. calculations use a 2024 GDP level of $29.2 trillion, while the global calculations use a 2024 global GDP level of approximately $111.0 trillion. The two sets of calculations hold fixed the full 2024–2027 reported capital-expenditure benchmark, annual real GDP growth, the diffusion path, the 20-year recovery horizon, and the assumed capital-bearing value-capture rate. The global-GDP calculations provide economic-scale comparisons. Their economic base is broader than the geographic coverage of the investment benchmark, AI-related output, and cash flow accruing to the firms in A. The reported values are the required long-run GDP effects associated with each assumed value of θ_A. Values are rounded to one decimal place.

Source: Author's calculations.

Table 5. Required Long-Run GDP Effects under Alternative Recovery Horizons

Recovery horizon	$\theta_A = 20\%$	$\theta_A = 10\%$	$\theta_A = 5\%$	$\theta_A = 2.5\%$	$\theta_A = 1\%$
10 years	14.6%	29.2%	58.3%	116.7%	291.6%
15 years	7.6%	15.2%	30.5%	61.0%	152.5%
20 years	5.5%	11.1%	22.1%	44.2%	110.5%
25 years	4.6%	9.2%	18.3%	36.6%	91.5%
50 years	3.3%	6.6%	13.1%	26.2%	65.6%
100 years	3.0%	6.0%	12.1%	24.2%	60.5%

Note*:* The calculations use U.S. GDP as the economic base, a real required return of 8 percent, the full 2024–2027 reported capital-expenditure benchmark, and the baseline logistic diffusion path. The capital-bearing value-capture rate, θ_A, is held constant over each recovery horizon. The reported values correspond to the GDP-only case, $\zeta = 0$, and are rounded to one decimal place.

Source*:* Author's calculations.

Table 6. Effects of Post-2027 Expansion Investment on Investment-Recovery Requirements

Case	Added PV of investment	Required cash-flow intensity	Required λ if 5% capture	Required λ if 2.5% capture
No post-2027 expansion	$0.00T	1.11%	22.1%	44.2%
Later expansion = 25% of PV of the 2024–2027 benchmark	$0.52T	1.38%	27.6%	55.2%
Later expansion = 50% of PV of the 2024–2027 benchmark	$1.05T	1.66%	33.2%	66.4%
Later expansion = 100% of PV of the 2024–2027 benchmark	$2.09T	2.21%	44.2%	88.4%

Note: The present value of additional investment is measured relative to the $2.091 trillion present value of the full 2024–2027 reported capital-expenditure benchmark. The calculations use unrounded values and assume a 20-year recovery horizon, a real required return of 8 percent, the baseline logistic diffusion path, and U.S. GDP as the economic base. The reported GDP effects correspond to the GDP-only case, $\zeta = 0$. Values are rounded to two decimal places for investment amounts and required cash-flow intensity and to one decimal place for required long-run GDP effects.

Source*:* Author's calculations.

Appendix A. Numerical Calculation of the Required Cash-Flow Intensity

This appendix reports the numerical construction of the required cash-flow intensity used in the valuation results. Under the GDP-only specification, the required cash-flow intensity over a recovery horizon T is

$$H_T(r) \equiv \frac{I_0^A(r)}{\sum_{t=1}^{T}(1+r)^{-t} D_t Y_t^0},$$

where D_t is the logistic diffusion index defined in Section 2 and Y_t^0 is GDP along the no-AI path. The corresponding infinite-horizon value is

$$H_\infty(r) \equiv \lim_{T\to\infty} H_T(r) = \frac{I_0^A(r)}{\sum_{t=1}^{\infty}(1+r)^{-t} D_t Y_t^0}.$$

Because D_t converges to one and $Y_t^0 = Y_0^0(1+g)^t$, the infinite sum is finite when $r > g$, as in the calibration. The numerical calculations date the 2024 capital expenditure at $t = 0$ and include GDP-related value flows from $t = 1$ through the end of the recovery horizon.

Under the GDP-only break-even condition, the required long-run GDP effect is

$$\lambda_T = \frac{H_T(r)}{\theta_A}, \ \lambda_\infty = \frac{H_\infty(r)}{\theta_A}.$$

For example, the 20-year calculation at an 8 percent required return implies a required long-run GDP effect of 22.1 percent when the capital-bearing value-capture rate is 5 percent, while the corresponding infinite-horizon value is 12.0 percent. The finite-horizon results under alternative capital-bearing value-capture rates and recovery horizons are reported in Tables 4 and 5.

Table A1. Numerical Calculation of the Required Cash-Flow Intensity

Recovery horizon	Real required return	PV of 2024–2027 investment	Discounted, diffusion-adjusted U.S. GDP base	Required cash-flow intensity
20 years	8%	$2.091T	$189.2T	1.105%
20 years	15%	$1.845T	$92.2T	2.000%
Infinite horizon	8%	$2.091T	$347.5T	0.602%

Note: The calculations use U.S. GDP as the economic base, starting from a 2024 GDP level of $29.2 trillion, annual real GDP growth of 2 percent, and the baseline logistic diffusion path with $k = 0.5$ and $t_0 = 7$ years. The 20-year calculations assign no terminal value. The infinite-horizon calculation includes all future diffusion-adjusted GDP flows under the maintained diffusion path and assumes no additional expansion investment after 2027. Dollar amounts are in trillions of constant 2024 dollars. Values are rounded as shown.

Source: Author's calculations.

Appendix B. Extension to Maintenance, Replacement, and Post-2027 Expansion Investment

B.1. General Adjusted Break-Even Condition

The baseline framework defines π_t^A as AI-attributable cash flow available for investment recovery after deducting maintenance and replacement investment. To make the accounting boundary explicit, this appendix first considers an alternative formulation in which maintenance and replacement investment is not deducted from cash flow and is instead included on the investment side of the break-even condition. The framework also allows for post-2027 expansion investment that lies outside the 2024–2027 reported capital-expenditure benchmark.

Let $\tilde{\pi}_t^A$ denote AI-attributable cash flow before deducting maintenance and replacement investment, but after the other deductions included in the valuation boundary. Let I_t^R denote maintenance and replacement investment not deducted from $\tilde{\pi}_t^A$, and let I_t^E denote post-2027 expansion investment. The baseline cash-flow measure satisfies

$$\pi_t^A = \tilde{\pi}_t^A - I_t^R \tag{B.1}$$

The present values of maintenance and replacement investment and post-2027 expansion investment over a recovery horizon T are

$$I_0^R(r) = \sum_{t=1}^{T}(1+r)^{-t}\, I_t^R \tag{B.2}$$

and

$$I_0^E(r) = \sum_{t=4}^{T}(1+r)^{-t}\, I_t^E \tag{B.3}$$

respectively. Because date 0 corresponds to the beginning of 2024, $t = 4$ corresponds to 2028, the first year after the 2024–2027 reported capital-expenditure benchmark.

Define the capital-bearing value-capture rate associated with the pre-replacement cash-flow measure as

$$\tilde{\theta}_A \equiv \frac{\sum_{t=1}^{T}(1+r)^{-t}\tilde{\pi}_t^A}{\sum_{t=1}^{T}(1+r)^{-t}S_t^{AI}} \tag{B.4}$$

When maintenance and replacement investment is treated on the investment side, the adjusted break-even condition is

$$\sum_{t=1}^{T}(1+r)^{-t}\,\tilde{\pi}_t^A \geq I_0^A(r) + I_0^R(r) + I_0^E(r) \tag{B.5}$$

or, equivalently,

$$\tilde{\theta}_A \sum_{t=1}^{T}(1+r)^{-t}\, S_t^{AI} \geq I_0^A(r) + I_0^R(r) + I_0^E(r) \tag{B.6}$$

Under the GDP-only specification, $S_t^{AI} = \lambda D_t Y_t^0$. Using the discounted, diffusion-adjusted GDP base $B_T(r)$ defined in Section 2, the adjusted break-even condition can therefore be written as

$$\tilde{\theta}_A \lambda B_T(r) \geq I_0^A(r) + I_0^R(r) + I_0^E(r) \tag{B.7}$$

Define the corresponding adjusted required cash-flow intensity as

$$\tilde{H}_T(r) \equiv \frac{I_0^A(r)+I_0^R(r)+I_0^E(r)}{B_T(r)} \tag{B.8}$$

The required long-run GDP effect under the pre-replacement cash-flow definition is then

$$\lambda \geq \frac{\tilde{H}_T(r)}{\tilde{\theta}_A} \quad \text{(B.9)}$$

B.2. Relation to the Baseline Cash-Flow Measure

The distinction between $\tilde{\pi}_t^A$ and π_t^A is an accounting distinction rather than a difference in the underlying investment-recovery condition. From equation (B.1),

$$\sum_{t=1}^{T}(1+r)^{-t}\,\pi_t^A = \sum_{t=1}^{T}(1+r)^{-t}\,\tilde{\pi}_t^A - I_0^R(r) \quad \text{(B.10)}$$

Substituting equation (B.10) into equation (B.5) yields

$$\sum_{t=1}^{T}(1+r)^{-t}\,\pi_t^A \geq I_0^A(r) + I_0^E(r) \quad \text{(B.11)}$$

Thus, maintenance and replacement investment cancels when the cash-flow and investment boundaries are adjusted consistently. Treating these outlays as a deduction from cash flow or as an additional investment requirement produces the same break-even condition.

The baseline framework uses the net cash-flow measure π_t^A, so maintenance and replacement investment is already reflected in the associated capital-bearing value-capture rate,

$$\theta_A \equiv \frac{\sum_{t=1}^{T}(1+r)^{-t}\pi_t^A}{\sum_{t=1}^{T}(1+r)^{-t}S_t^{AI}} \quad \text{(B.12)}$$

Accordingly, maintenance and replacement investment should not be added separately to the investment numerator under the baseline definition. When post-2027 expansion investment is also excluded, $I_0^E(r) = 0$, equation (B.11) reduces to the baseline break-even condition developed in Section 2,

$$\theta_A\lambda \geq H_T(r) \quad \text{(B.13)}$$

The accounting equivalence is useful for distinguishing maintenance and replacement investment from additional expansion investment. The former is already incorporated in the baseline cash-flow measure, whereas the latter lies outside both the baseline cash-flow measure and the 2024–2027 reported capital-expenditure benchmark.

B.3. Post-2027 Expansion-Investment Overlay

Table 6 retains the baseline cash-flow definition and therefore does not add a separate maintenance or replacement investment term. It considers only post-2027 expansion investment that lies outside the 2024–2027 reported capital-expenditure benchmark. Under this convention, equation (B.11) becomes

$$\sum_{t=1}^{T}(1+r)^{-t}\,\pi_t^A \geq I_0^A(r) + I_0^E(r) \quad \text{(B.14)}$$

Under the GDP-only specification, the corresponding required cash-flow intensity is

$$H_T^E(r) \equiv \frac{I_0^A(r)+I_0^E(r)}{B_T(r)} \quad \text{(B.15)}$$

and the required long-run GDP effect is

$$\lambda^E = \frac{H_T^E(r)}{\theta_A} \quad \text{(B.16)}$$

For Table 6, the present value of post-2027 expansion investment is parameterized as a fraction ϕ of the present value of the 2024–2027 reported capital-expenditure benchmark:

$$I_0^E(r) = \phi I_0^A(r) \tag{B.17}$$

It follows that

$$H_T^E(r) = (1+\phi)H_T(r) \tag{B.18}$$

and, holding θ_A and $B_T(r)$ fixed,

$$\lambda^E = (1+\phi)\lambda \tag{B.19}$$

Table 6 reports $\phi = 0.25, 0.50$, and 1.00. Thus, under the maintained assumptions, post-2027 expansion investment with a present value equal to 25, 50, or 100 percent of the baseline investment benchmark raises both the required cash-flow intensity and the required long-run GDP effect by the same respective proportions. The calculation holds the discounted, diffusion-adjusted GDP base and the capital-bearing value-capture rate fixed and does not specify a separate additional value path attributable to the post-2027 expansion investment. It therefore isolates the effect of increasing the investment amount that must be recovered.